RELAXATION PROCESSES IN ELECTRODYNAMIC PLASMA ACCELERATION

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Relaxation processes which occur in a plasma during its electrodynamic acceleration are analyzed. The corresponding mass-transfer equations are solved with an account of Joule erosion of the electrodes, particle diffusion, charge exchange, and the delay of mass separation.

An extremely wide variety of physical processes lead to mass transfer in an accelerated plasma [1-11]. These processes are distinguished by their intensity and rapidity. The time required for the development of an entire process may be of the order of tens of microseconds, and sometimes even less, and the large energies available in a pulsed discharge cause intense mass transfer. Acting simultaneously are a wide variety of physical phenomena resulting in the addition of matter to the plasma, and there are complex relationships among the electromechanical processes and phenomena accompanying electrodynamic plasma acceleration.

We will take up the most important of the physical processes which lead to intense mass transfer. These elementary processes were analyzed in detail in [1-4], where methods were described for their phenomenological and kinetic description. Here we will give only the necessary equations, referring the reader to [1-5] for the details.

One process which tends to increase the mass during its acceleration is ionization. We can distinguish between bulk ionization of a solid and surface ionization at the accelerating electrodes. Ionization may result from collisions between particles, heating, the effects of photons, or an increase in the pressure. The ionization process is characterized by ionization cross sections. The kinetic equation for ionization is [4]

$$\frac{\partial n}{\partial t} = \omega_i n. \tag{1}$$

A competing process, which leads to plasma deionization, is recombination. We can distinguish: bulk recombination; surface recombination; recombination according to the type of interaction of the particles, e.g., electron–ion recombination; and recombination according to the type of interacting particles, e.g., two-particle or three-particle recombination. The kinetic equation for two-particle recombination is [4]

$$\frac{\partial n}{\partial t} = -\rho n^2. \tag{2}$$

For three-particle recombination, we have [5]

$$\frac{\partial n}{\partial t} = -\alpha_3 n_e^3. \tag{3}$$

Charge exchange involves the exchange of an electron between a neutral atom and an ion and leads to mass transfer during plasma acceleration. If the ionization potentials are approximately equal, this is a very efficient process.

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Attachment (or capture) in the case of two-particle collisions is described by the kinetic equation

$$\frac{\partial n}{\partial t} = -h\nu n, \tag{4}$$

while in the case of three-particle collisions it is described by [4]

$$\frac{\partial n}{\partial t} = -h_1 \nu_1 n^2. \tag{5}$$

Diffusion leads to a redistribution of particles in inhomogeneous media. When there are particles differing in charge, a special type of diffusion occurs: ambipolar diffusion, due to the effect of the intrinsic electromagnetic fields. The corresponding kinetic equation is

$$\frac{\partial n}{\partial t} = D \operatorname{div} \operatorname{grad} n. \tag{6}$$

Ambipolar diffusion can be thought of as a process in which electrons, diffusing rapidly from a plasma, attract positive ions by means of the electrostatic fields due to the charge separation. As was mentioned in [5], there is a transition region between free diffusion and ambipolar diffusion which spans many orders of magnitude in the electron density; the ambipolar-diffusion coefficient is given by

$$D_a = \frac{D_+ \mu_e + D_e \mu_+}{\mu_+ + \mu_e}. \tag{7}$$

When ambipolar diffusion predominates over other types of diffusive loss of electrons and ions, the coefficient D in Eq. (6) can be replaced by D_a:

$$\frac{\partial n}{\partial t} = D_a \operatorname{div} \operatorname{grad} n. \tag{8}$$

If diffusion plays the primary role, Eq. (8) can be rewritten approximately as

$$\frac{\partial n}{\partial t} = -\frac{D_a}{\lambda_D^2} n = -\frac{n}{\tau_D}. \tag{9}$$

Mass transfer occurs between the plasma and surfaces because of particle emission from the solid and because of surface adsorption processes. We can distinguish between thermionic emission of particles, field emission (or cold emission) under the influence of applied fields, photoemission, etc. Mass-transfer processes also occur in an electrode layer as a result of phase conversions. The surface of an electrode can be destroyed as a result of its interaction with particles, as a result of its melting by Joule heating (Joule melting), etc. Two erosion mechanisms are known: in the first, mass is liberated by electrode erosion caused by ion bombardment (cathode sputtering), in which case we have

$$\Delta m = \int_0^t I\,dt \tag{10}$$

in the second, Joule melting of the electrode occurs, and we have

$$\Delta m \sim \int_0^t I^2\,dt. \tag{11}$$

Phenomena occurring near the electrode are extremely complicated and have been studied relatively little. We might also mention the so-called elementary processes leading to mass transfer, e.g., various types of secondary ionization, emission, and recombination.

Taking into account ambipolar diffusion and ionization of the particles in the accelerated plasma, the increase in the plasma mass due to electrode erosion caused by ion bombardment, charge exchange, and attachment of electrons to ions; and neglecting terms which are nonlinear in the mass, e.g., two-particle and three-particle recombination; we can write the mass-balance equation as [6]

$$\frac{dm}{dt} + a_1 m = a_3 F(t), \tag{12}$$

where a_1 and a_3 are proportionality factors, which are determined from experiment or theoretically, on the basis of the kinetics of the elementary processes. The coefficient a_1 describes ionization and diffusion, while a_3 describes cathode sputtering (a_1 may be either positive or negative). Also, F(t) is some function which takes into account the external mass sources. Using

$$\mu=\frac{m}{m_0},\quad \gamma_1=a_1\sqrt{L_0C_0},\quad \gamma_3=\frac{a_3C_0V_0}{m_0},\quad \tau=\frac{t}{\sqrt{L_0C_0}}, \tag{13}$$

we can rewrite Eq. (12) in dimensionless form:

$$\frac{d\mu}{d\tau}+\gamma_1\mu=\gamma_3 f(\tau). \tag{14}$$

Assuming that we have $\mu = 1$ when $\tau = 0$, we find the following solution of Eq. (14):

$$\mu(\tau)=\exp(-\gamma_1\tau)+\gamma_3\exp(-\gamma_1\tau)\int_0^\tau f(\tau)\exp(\gamma_1\tau)\,d\tau. \tag{15}$$

The oscillogram of the dimensionless discharge φ' can be approximated sufficiently accurately by

$$\varphi'(\tau)=\exp\left(-\frac{\alpha\tau}{2}\right)\sin\tau. \tag{16}$$

Assuming that we have in (14)

$$f(\tau)=\left|\exp\left(-\frac{\alpha\tau}{2}\right)\sin\tau\right|, \tag{17}$$

which corresponds to cathode sputtering [Eq. (10)], we can write Eq. (15) as

$$\mu(\tau)=\exp(-\gamma_1\tau)+\gamma_3\exp(-\gamma_1\tau)\int_0^\tau \exp\left[\left(-\frac{\alpha}{2}+\gamma_1\right)\tau\right]|\sin\tau|\,d\tau$$

$$=\exp(-\gamma_1\tau)+\frac{\gamma_3}{\left(\gamma_1-\frac{\alpha}{2}\right)^2+1}\left\{1+\exp\left(-\frac{\alpha\tau}{2}\right)\left[\left(\gamma_1-\frac{\alpha}{2}\right)\sin\tau-\cos\tau\right]\right\}. \tag{18}$$

Using (11) and (16), we can describe mass transfer due to electrode melting caused by Joule heating by

$$\frac{d\mu}{d\tau}+\gamma_1\mu=\gamma_4\exp(-\alpha\tau)\sin^2\tau, \tag{19}$$

which has the following solution under the initial conditions $\tau = 0$, $\mu = 1$:

$$\mu(\tau)=\exp(-\gamma_1\tau)\left[1-\frac{\gamma_4}{2(\gamma_1-\alpha)}+\frac{(\gamma_1-\alpha)\gamma_4}{2(\gamma_1-\alpha)^2+8}\right]+\gamma_4\exp(-\alpha\tau)$$
$$\times\left\{\frac{1}{2(\gamma_1-\alpha)}-\frac{1}{(\gamma_1-\alpha)^2+4}\left[\frac{(\gamma_1-\alpha)}{2}\cos 2\tau+\sin 2\tau\right]\right\}. \tag{20}$$

To describe relaxation more completely, we modify Eq. (14) and write it in more general form:

$$\frac{d^2\mu}{d\tau^2}+\frac{1}{\Delta\tau}\frac{d\mu}{d\tau}+\frac{\gamma_1}{\Delta\tau}\mu=\frac{\gamma_4}{\Delta\tau}f(\tau). \tag{21}$$

We have added a term proportional to the second derivative of the mass with respect to time. Comparing this equation with the Newtonian dynamic equations, we see that this second derivative takes into account the time lag corresponding to mass liberation [10]. The coefficient γ_1 can be positive, corresponding to a decrease in mass, or negative (corresponding to an increase in mass).

The solution of this equation is:

a) for $\lambda^2=\frac{1}{\Delta\tau^2}-4\frac{\gamma_1}{\Delta\tau}>0$

$$\mu(\tau)=C_1\exp\left[\frac{\left(\lambda-\frac{1}{\Delta\tau}\right)\tau}{2}\right]+C_2\exp\left[\frac{\left(-\frac{1}{\Delta\tau}-\lambda\right)\tau}{2}\right]$$
$$+\frac{2}{\lambda}\int_0^\tau \gamma_4 f(\tau')\exp\left[\frac{1}{2\Delta\tau}(\tau'-\tau)\right]\operatorname{sh}\left[\frac{\lambda}{2}(\tau'-\tau)\right]d\tau'; \tag{22}$$

b) for $\varepsilon^2 = 4\frac{\gamma_1}{\Delta\tau} - \frac{1}{\Delta\tau^2} > 0$,

$$\mu(\tau) = \exp\left(-\frac{\tau}{2\Delta\tau}\right)\left(C_1\cos\frac{1}{2}\varepsilon\tau + C_2\sin\frac{1}{2}\varepsilon\tau\right)$$

$$+\frac{2}{\varepsilon}\int_0^\tau \gamma_4 f(\tau')\exp\left[\frac{1}{2\Delta\tau}(\tau'-\tau)\right]\sin\frac{\varepsilon}{2}(\tau-\tau')\,d\tau'; \quad (23)$$

c) for $\frac{1}{\Delta\tau^2} = 4\frac{\gamma_1}{\Delta\tau}$

$$\mu(\tau) = \exp\left(-\frac{\tau}{2\Delta\tau}\right)(C_1\tau + C_2) + \int_0^\tau \gamma_4 f(\tau)(\tau-\tau')\exp\left[\frac{1}{2\Delta\tau}(\tau'-\tau)\right]d\tau'. \quad (24)$$

The arbitrary constants C_1 and C_2 in Eqs. (22)-(24) are determined from the initial conditions.

Adopting the approximation corresponding to a Joule melting of the electrodes,

$$f(\tau) = \exp(-\alpha\tau)\sin^2\tau, \quad (25)$$

and assuming $\lambda^2 > 0$ in (22), we find, for the initial conditions

$$\mu = 1, \quad \frac{d\mu}{d\tau} = 0 \quad \text{at} \quad \tau = 0, \quad (26)$$

the following solution of Eq. (21):

$$\mu(\tau) = M - N + C_1\exp(b\tau) + C_2\exp(-d\tau) + \left[N - M\cos 2\tau + \frac{(a^2-c^2)\sin 2\tau}{\lambda\Delta\tau(a^2+4)(c^2+4)}\right]\exp(-\alpha\tau), \quad (27)$$

where

$$C_1 = 1 - C_2; \quad N = \frac{\gamma_4}{2ac\,\Delta\tau}; \quad \lambda = \pm\sqrt{\frac{1}{\Delta\tau^2} - 4\frac{\gamma_1}{\Delta\tau}}; \quad a = \frac{1}{2\Delta\tau} - \frac{\lambda}{2} - \alpha;$$

$$c = \frac{1}{2\Delta\tau} + \frac{\lambda}{2} - \alpha; \quad b = \frac{\lambda}{2} - \frac{1}{2\Delta\tau}; \quad d = \frac{\lambda}{2} + \frac{1}{2\Delta\tau};$$

$$C_2 = \frac{b}{\lambda} - \gamma_4\frac{\alpha\lambda(a^2+4)(c^2+4) - ac[(\alpha a-4)(c^2+4) + (4-\alpha c)(a^2+4)]}{2ac\lambda^2\Delta\tau(a^2+4)(c^2+4)};$$

$$M = \gamma_4\frac{a(c^2+4) - c(a^2+4)}{2\lambda\Delta\tau(a^2+4)(c^2+4)}.$$

From Eq. (27) we see that by varying $\Delta\tau$ we can adjust the time lag of the mass liberation. If we eliminate the third term on the left side of Eq. (21), i.e., set $\gamma_1 = 0$, and assume $\gamma_4 = 1$, we can write the solution of the equation

$$\frac{d^2\mu}{d\tau^2} + \frac{1}{\Delta\tau}\cdot\frac{d\mu}{d\tau} = \frac{1}{\Delta\tau}f(\tau) \quad (28)$$

in a simpler form:

$$\mu(\tau) = C_1 + C_2\exp\left(-\frac{\tau}{\Delta\tau}\right) + \int_0^\tau f(\tau')\left[1 - \exp\left(\frac{\tau'-\tau}{\Delta\tau}\right)\right]d\tau'$$

$$= 1 + \frac{\alpha+\beta}{2\alpha\beta} - \frac{\alpha(\beta^2+4) + \beta(\alpha^2+4)}{2(\alpha^2+4)(\beta^2+4)} - \frac{\Delta\tau\alpha(\beta^2+4) + \Delta\tau\beta(4-\alpha\beta)}{2\beta(\beta^2+4)}$$

$$+ \frac{\Delta\tau\alpha(\beta^2+4) + \Delta\tau\beta(4-\alpha\beta)}{2\beta(\beta^2+4)}\exp\left(-\frac{\tau}{\Delta\tau}\right)$$

$$-\left\{\frac{\alpha+\beta}{2\alpha\beta} - \frac{[\beta(\alpha^2+4) + \alpha(\beta^2+4)]\cos 2\tau}{2(\alpha^2+4)(\beta^2+4)} - \frac{(\alpha^2-\beta^2)\sin 2\tau}{(\alpha^2+4)(\beta^2+4)}\right\}\exp(-\alpha\tau), \quad (29)$$

where $\beta = (1/\Delta\tau) - \alpha$.

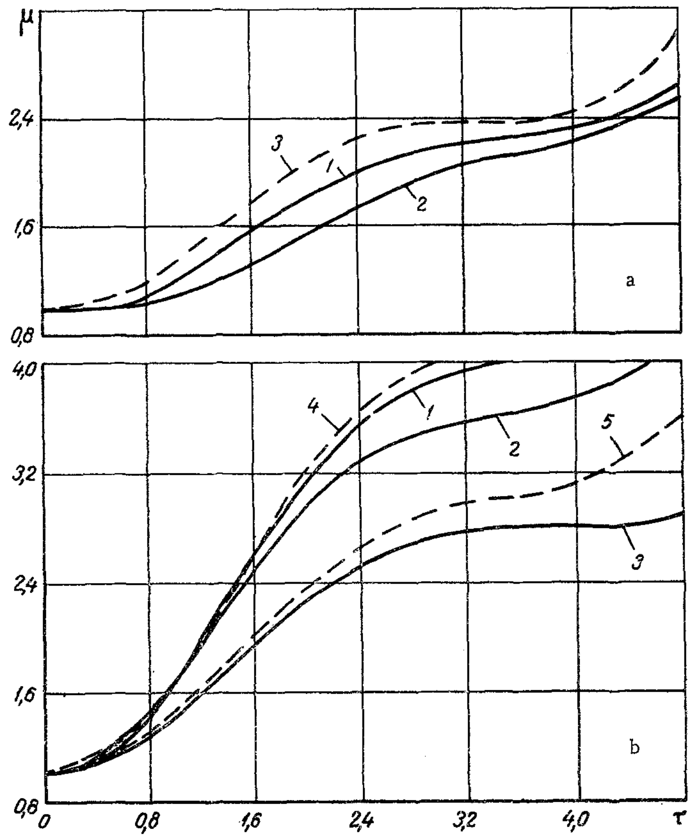

Fig. 1. Dependence of the mass μ of the accelerated plasma on the time τ.

Figure 1a (curves 1 and 2) show the behavior of the mass during electrodynamic plasma acceleration; curve 1 corresponds to Eq. (29) for $\alpha = 0.1$ and $\Delta\tau = 0.5$, while curve 2 corresponds to $\alpha = 0.1$ and $\Delta\tau = 1.0$. We see that a change in $\Delta\tau$ shifts the maximum mass evolution. Curve 3 is found by solving the equation

$$\frac{d\mu}{d\tau} = \gamma_4 \exp(-\alpha\tau)\sin^2\tau \tag{30}$$

for the parameter values $\gamma_4 = 1$ and $\alpha = 0.1$.

A detailed study was made in [6] of the combined effects on acceleration of plasma recombination, ambipolar diffusion, electrode erosion, charge exchange, attachment of electrons to ions, and resistive forces. Curves 1-3 in Fig. 1b show the solution of the system of equations describing electrodynamic plasma acceleration with an account of the "elementary" processes cited above [6]:

$$\frac{dy}{d\tau} = y', \tag{31}$$

$$\frac{dy'}{d\tau} = \frac{q}{\mu}\varphi'^2 - \frac{y'}{\mu}(\delta_1 + \delta_2\mu + \delta_3|\varphi'| + \delta_4 y') - \frac{y'}{\mu}(-\gamma_1\mu - \gamma_2\mu^2 + \gamma_3|\varphi'| + \gamma_4\varphi'^2), \tag{32}$$

$$\frac{d\varphi}{d\tau} = -\varphi', \tag{33}$$

$$\frac{d\varphi'}{d\tau} = \frac{\varphi - \alpha\varphi' - y'\varphi'}{1+y}, \tag{34}$$

$$\frac{d\mu}{d\tau} = -\gamma_1\mu - \gamma_2\mu^2 + \gamma_3|\varphi'| + \gamma_4\varphi'^2, \tag{35}$$

where

$$\mu = \frac{m}{m_0};\quad \varphi = \frac{V}{V_0};\quad \tau = \frac{t}{\sqrt{L_0 C_0}};\quad y = \frac{b}{L_0}z;\quad y' = b\sqrt{\frac{C_0}{L_0}}\,v;$$

$$\varphi' = \sqrt{\frac{L_0}{C_0}}\frac{I}{V_0}; \quad q = \frac{b^2 C_0^2 V_0^2}{2m_0 L_0}; \quad \alpha = R\sqrt{\frac{C_0}{L_0}};$$
$$\gamma_1 = a_1\sqrt{L_0C_0}; \quad \gamma_2 = a_2 m_0\sqrt{L_0C_0}; \quad \gamma_3 = \frac{a_3 C_0 V_0}{m_0};$$
$$\gamma_4 = \frac{C_0^2 V_0^2}{m_0\sqrt{L_0C_0}}a_4; \quad \delta_1 = \frac{b_1}{m_0}\sqrt{L_0C_0}; \quad \delta_2 = b_2\sqrt{L_0C_0};$$
$$\delta_3 = \frac{1}{2}\frac{m_i}{e}\frac{C_0V_0}{m_0}; \quad \delta_4 = \frac{b_4L_0}{bm_0}. \tag{36}$$

The values and physical meaning of the dimensionless parameters q, α, γ_i, and δ_j and the dimensionless variables τ, y, y', φ, φ', and μ were discussed in [6]. System (31)-(36) was solved numerically by the Runge–Kutta method with a τ integration step of h = 0.2 and under the initial conditions

$$\text{for } \tau = 0 \quad \mu = \varphi = 1, \quad \varphi' = y' = y = 0. \tag{37}$$

Curve 1 corresponds to q = 1, α = 0.1, $\gamma_1 = \gamma_2 = \delta_1 = \delta_3 = \delta_4 = 0$, $\gamma_3 = \gamma_4 = 1$, $\delta_2 = 1$; curve 2 corresponds to q = 1, α = 0.1, $\gamma_1 = \gamma_2 = \delta_1 = \delta_3 = \delta_4 = 0$, $\gamma_3 = \gamma_4 = 1$, $\delta_2 = 0.1$; and curve 3 corresponds to q = 1, α = 0.1, $\gamma_1 = \gamma_2 = \delta_1 = \delta_3 = \delta_4 = 0$, $\gamma_3 = 1$, $\gamma_4 = 0.1$, $\delta_2 = 0.1$.

We note that the dimensionless parameters γ_1 and δ_2 are functions of the coefficients a_1 and b_2 and of the time

$$\delta\tau = \frac{1}{\sqrt{L_0C_0}}, \tag{38}$$

i.e., using (36) and (38), we can write these parameters as

$$\gamma_1 = \frac{a_1}{\delta\tau}, \quad \delta_2 = \frac{b_1}{\delta\tau}. \tag{39}$$

Curve 1 in Fig. 1b corresponds to intense electrode erosion due to Joule melting and ion bombardment ($\gamma_3 = \gamma_4 = 1$) in the case in which diffusive friction is important ($\delta_2 = 1$). A tenfold decrease in δ_2 (curve 2), which may be thought of as a tenfold increase in $\delta\tau$ in (39), leads to a time delay of the mass liberation, which leads in turn to an increase in y' [6]. Curve 3, corresponding to a value of γ_4 one-tenth of that corresponding to curve 2, corresponds to a more intense mass liberation due to ion bombardment with these parameters. Figure 1b (curves 4, 5) also shows the results found from solving

$$\frac{d\mu}{d\tau} = \gamma_3 \exp\left(-\frac{\alpha\tau}{2}\right)|\sin\tau| + \gamma_4 \exp(-\alpha\tau)\sin^2\tau. \tag{40}$$

Assuming that μ = 1 when τ = 0, we find the following solution for Eq. (40):

$$\mu(\tau) = 1 + \frac{\gamma_3\left[4 - 2\exp\left(-\frac{\alpha\tau}{2}\right)(\alpha\sin\tau + 2\cos\tau)\right]}{\alpha^2+4} + \frac{\gamma_4\{4 - \exp(-\alpha\tau)[\alpha^2+4+\alpha(2\sin 2\tau - \alpha\cos 2\tau)]\}}{2\alpha(\alpha^2+4)}. \tag{41}$$

Curve 4 corresponds to parameters α = 0.1,and $\gamma_3 = \gamma_4 = 1$, while curve 5 corresponds to α = 0.1, $\gamma_3 = 1$, and $\gamma_4 = 0.1$.

The discrepancy between curves 1 and 4 and that between curves 3 and 5 (Fig. 1b) results from the error in the approximation of the discharge current, which can be evaluated from Fig. 2. This error occurs because approximation (16) for φ' neglects the effect of the accelerated motion on the current; this motion reduces the current because magnetic energy is transformed into kinetic energy of the plasma. Accordingly, as we can see from Fig. 2, approximation (16) with α = 0.1 yields large currents (curve 4) and thus, according to (40) leads to an increase in the mass, as is seen in Fig. 1b.

Curves 1-3 in Fig. 2 are the result of a numerical solution of Eqs. (31)-(35) under initial conditions (37); the values of the parameters q, α, γ_i, and δ_j are those corresponding to curves 1-3 in Fig. 1b.

In the integration of Eqs. (14) and (40) with approximation (17), which is not an analytic function, the magnitude of the mass-liberation increment in Eqs. (18) and (41) should be evaluated by changing the sign of the discharge current to negative. Equations (18) and (41) were derived without an account of this remark.

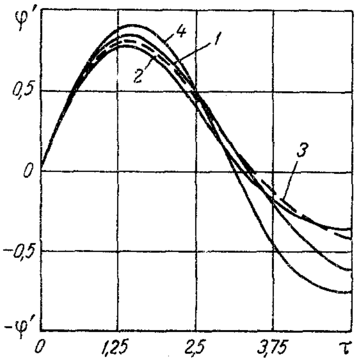

Fig. 2. The current φ' as a function of the time τ, and comparison with approximation (16).

Comparing the results calculated from complete system (31)-(35) describing electrodynamic plasma acceleration with those found by our approximate method, we find a good agreement; this method significantly simplifies and shortens the study of these topics.

Account of the nonlinear terms describing such processes as two-particle recombination (2) and three-particle combination (3) leads to the need to solve nonlinear equation (35) and its generalization with an account of the inertial term, analogous to Eq. (21). We will report separately the use of various asymptotic methods [14, 15] to analyze these equations.

In conclusion we wish to point out a relation between Eq. (21) and the equations which describe relaxation in a viscoelastic element in the theory of creep and the equations taking into account relaxation of the polarization **P** and magnetization **M** in linear dispersive electric and magnetic media. For a viscoelastic element we can write the general form of the differential dependence of the strain ψ on the time t as [13]

$$\frac{d^2\psi}{dt^2} + C_2\frac{d\psi}{dt} + C_1\psi = Q(t). \tag{42}$$

To describe the relaxation of **P** and **M** we can write [12]

$$\frac{d^2\mathbf{P}}{dt^2} + k_2\frac{d\mathbf{P}}{dt} + k_1\mathbf{P} = G(t), \tag{43}$$

$$\frac{d^2\mathbf{M}}{dt^2} + l_2\frac{d\mathbf{M}}{dt} + l_1\mathbf{M} = U(t). \tag{44}$$

Comparing Eq. (21) with Eqs. (42)-(44), we immediately perceive their similar structure; we can accordingly use the same methods to study these different processes.

NOTATION

ω_i	is the ionization probability;
n	is the density of the ionized particles;
t	is the time;
ρ	is the recombination coefficient;
α_3	is the coefficient for three-particle recombination;
h	is the attachment coefficient;
ν	is the collision frequency;
D	is the diffusion coefficient;
D_e and D_+	are the diffusion coefficients for electrons and ions;
μ_e and μ_+	are the mobilities;
$\tau_D = \lambda_D^2/D_a$	is the diffusive-relaxation time;
λ_D	are the characteristic dimensions;
I	is the discharge current in the external circuit;
m	is the mass of accelerated plasma;
m_0	is the initial mass of accelerated plasma;
L_0	is the initial circuit inductance;
C_0	is the capacitance of capacitor bank;
V_0	is the bank voltage;
α	is the dimensionless circuit resistance;
C_1, C_2, k_1, k_2, l_1, and l_2	are the corresponding proportionality factors;
Q(t)	is the function taking into account external voltage sources affecting the viscoelastic elements;
G(t) and U(t)	are functions taking into account the external sources affecting the media;
V	is the voltage;
R	is the resistance of the total circuit;
b	is the distributed inductance per unit length of the accelerator;

q, γ_i, and δ_j	are the dimensionless parameters;
a_1, a_2, a_3, and a_4	are the mass coefficients of diffusion, recombination, electrode erosion caused by ion bombardment, and electrode erosion due to Joule melting, respectively;
b_1, b_2, b_3, and b_4	are the proportionality factors taking into account the friction of the moving plasma with the electrode (b_1), friction during mass transfer (b_2, b_3), and the effect of the resistance of the external medium (b_4);
τ, y, y', φ, φ', and μ	are the dimensionless variables of time, coordinate, velocity, voltage, current, and mass.

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